PHYSICAL REVIEW D VOLUME 15, NUMBER 12 15 JUNE 1977

Interpretation of pseudoparticles in physical gauges*

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(Received 11 October 1976; revised manuscript received 21 March 1977)

The effects of pseudoparticle solutions to Yang-Mills field theories are discussed for gauges where the $F_{\mu\nu}$ uniquely determine the potentials. Even though a unique vacuum is found, the physical consequences obtained by previous authors are shown to occur. It is then shown how the $A_0 = 0$ gauge may be reformulated to give a unique vacuum. The many vacuums found by previous authors are seen to be artifacts of a particular parametrization of function space.

Recently 't Hooft[1] suggested that the pseudoparticle solution[2] to the four-dimensional Yang-Mills theory might provide a mechanism for the spontaneous breakdown of chiral invariance without requiring a "ninth" Goldstone boson. Subsequently, Jackiw and Rebbi[3] (JR) and Callan, Dashen, and Gross[4] (CDG) showed how this could be understood in terms of multiple-vacuum states; furthermore, their description of the theory leads rather naturally to the introduction of a CP-violating angle θ. However, their multiple-vacuum description is developed in the gauge $A_0 = 0$ and does not have an obvious extension to what we shall refer to as "physical" gauges, i.e., those in which the A_μ are uniquely determined by the $F_{\mu\nu}$.[5] Since the $A_0 = 0$ gauge has a number of subtle properties, such as the requirement that one impose a subsidiary condition, one might wonder whether the multiple vacuums and, consequently, the possibility of CP violation are artifacts of the gauge. Here we consider the problem of extending the analysis to physical gauges. Our analysis of these gauges leads to a unique vacuum, i.e., given the Lagrangian (or the corresponding Hamiltonian) there is a unique ground state. Despite this, the absence of a "ninth" Goldstone boson can be understood. Further, we find that by changing the Lagrangian it is possible to generate a family of CP-violating theories which agree with the usual Yang-Mills theory at the classical level, but not in quantum mechanics. The physical consequences of our formulation are thus identical with the JR-CDG picture. We then return to the $A_0 = 0$ gauge and show that it is possible to reformulate the theory in this gauge in such a manner that the multiple vacuums do not appear. The appearance of multiple vacuums is shown to be a result of having chosen to parametrize function space in a particular manner.

We have defined physical gauges by the requirement that the A_μ be uniquely determined by the $F_{\mu\nu}$. Therefore, in such a gauge the classical ground state, $F_{\mu\nu} = 0$ for all space, has a unique A_μ; there is no evidence for multiple vacuums at the classical level. Now let us consider the Euclidean functional-integral expression for the vacuum-to-vacuum transition amplitude

$$\langle 0|e^{-Ht}|0\rangle \underset{t\to\infty}{\longrightarrow} \int [dA][d\phi]\exp\left[-\int d^4x\,\mathcal{L}(A,\phi)\right] \times \delta(\ )\det(\ ), \tag{1}$$

where ϕ represents all the nongauge fields in the theory[6] and where the Faddeev-Popov δ function and determinant have been schematically indicated. The integral is to run over all fields which take the value $A_\mu = 0$ at Euclidean $t = \pm\infty$. The only requirements at spatial infinity are the vanishing of $F_{\mu\nu}$ and whatever boundary conditions are included in the specification of the gauge. Consequently, all topological classes of gauge fields are included. In particular, the presence of pseudoparticles will be evidenced in the axial gauge by nontrivial behavior at $z = \infty$. One can now calculate the vacuum expectation value of a string of chiral operators $\sigma_\pm = \bar{\Psi}(1\pm\gamma_5)\Psi$. Because all topological classes have been included, these matrix elements will satisfy the cluster decomposition property; there is no need to introduce multiple vacuums.

How then can we recover the physical consequences of the many-vacuum picture? First consider the question of the "ninth" Goldstone boson. If the theory contains massless fermions, a divergenceless axial-vector current can be defined by

$$\tilde{j}_5^\mu \equiv j_5^\mu - j_A^\mu, \tag{2}$$

where j_5^μ is the naive axial-vector current whose divergence is given by the Adler-Bell-Jackiw[7] anomaly, and j_A^μ is defined by

$$j_A^\mu = \frac{1}{4\pi^2}\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}\{A_\nu\partial_\alpha A_\beta + \tfrac{2}{3}A_\nu A_\alpha A_\beta\} \tag{3}$$

so that

$$\partial_\mu j_A^\mu = \frac{1}{16\pi^2}\mathrm{Tr}F_{\mu\nu}\tilde{F}^{\mu\nu}. \tag{4}$$

Although $\tilde{j}_5^\mu$ is divergenceless, the corresponding charge is not conserved,[8] since

$$\frac{d}{dt}\tilde{Q}_5 \equiv \frac{d}{dt}\int d^3x\, \tilde{j}_5^0 = -\int d^3x\, \partial_i \tilde{j}_5^i \neq 0. \qquad (5)$$

The final surface integral need not vanish; the long-range behavior of the potentials in the presence of pseudoparticles is such that j_A^i gives a finite contribution to the surface integral. Since there is no conserved charge, the Goldstone theorem does not apply.

Now consider changing the theory by adding a potentially CP-violating term of the form

$$\frac{\beta}{32\pi^2}\mathrm{Tr}F_{\mu\nu}\tilde{F}^{\mu\nu} = 2\beta\,\partial_\mu j_A^\mu \qquad (6)$$

to the Lagrangian density. This will have no effect on the classical field theory, since the addition of a total divergence does not change the classical equations of motion.[9] Furthermore, the Feynman rules are unchanged, so no CP violation will be seen in quantum perturbation theory. It is only because of the pseudoparticle, a nonperturbative quantum effect, that there is a possibility of CP violation.

The term we have added appears in CDG as part of an effective Lagrangian for a particular vacuum sector; however, it enters our description in a rather different manner. In the CDG description, there are many vacuums, with corresponding sectors. In the present development, there is a family of different theories which give the same results both at the classical level and in perturbation theory, but which differ when the possibility of pseudoparticles is taken into account. If this variety of families were introduced into the JR-CDG description, the effect would be that the θ sector of the $\beta = 0$ theory would be like the $\theta - \theta'$ sector of the $\beta = \theta'$ theory.

We thus have two rather different descriptions of the theory which lead to the same observable effects. Before discussing the relationship between these descriptions, it is perhaps useful to consider as a simple example the Lagrangian

$$L = \tfrac{1}{2}\dot{x}^2 + \lambda \cos x. \qquad (7)$$

If the range of x is taken to be $-\infty < x < \infty$, this describes a particle in a periodic potential. The states fall into sectors labeled by a parameter β with range $0 \le \beta < 2\pi$. The energy spectrum is a series of bands.

Of course, it is also possible to view this as the Lagrangian for a pendulum. In this case, the points x and $x + 2\pi n$ must be identified. The energy spectrum is now discrete. However, it is possible to add a term $b\dot{x}$ to the Lagrangian. Although this term has no effect on the classical equations of motion, it will shift the energy levels of the quantum theory. Each value of b leads to a spectrum equal to that of a particular sector of the periodic potential. There is thus a family of pendulum theories, each corresponding to a particular sector of the periodic-potential theory. [Note that instead of adding a term to the Lagrangian, it is possible to change the theory by requiring $\psi(2\pi) = e^{i\alpha}\psi(0)$ rather than $\psi(2\pi) = \psi(0)$; the result is the same.]

Let us now return to the Yang-Mills theory. We begin by recalling the difference in the canonical treatment of the two types of gauges. In the $A_0 = 0$ gauge, A_i and $\pi_i = F_{0i}$, $i = 1, 2, 3$, are all independent canonical fields; in physical gauges, the gauge condition and the constraint (Gauss's law) are used to eliminate both A_0 and some additional fields from the set of canonical variables. Thus, the $A_0 = 0$ gauge has "more" canonical variables; stated differently, the field strengths at any one time do not uniquely determine the corresponding potentials. Consequently, one must impose a subsidiary condition (Gauss's law). However, as JR and CDG noted, there is still an ambiguity arising from the possibility of gauge transformations which do not vanish at spatial infinity. In physical gauges, all such ambiguity is eliminated by the gauge conditions.

Now let us ask how the path integral is to be calculated. The general rule is that by integrating over all paths which run from a particular initial configuration to a particular final configuration, and then going to the limit of infinite time, information about vacuum matrix elements can be extracted. The application of this prescription in physical gauges is clear. In the $A_0 = 0$ gauge, however, one must decide whether "configuration" in this prescription is to be defined by the A_i or by the F_{ij}. The former is the choice made by CDG and JR; however, this method of parametrizing function space necessitates the introduction of many vacuums. In order to motivate an alternative parametrization, consider the paths shown schematically in Fig. 1. In a physical gauge [Fig. 1(a)], all three paths begin and end at the same point. In the $A_0 = 0$ gauge we may use the freedom to perform time-independent gauge transformations to require that they begin at the same point [Fig. 1(b)] or that they be in a particular "winding-number" sector at time $t = 0$ [Fig. 1(c)], but it is impossible to require that all three paths both begin and end at the same point. If we want the $A_0 = 0$ path integral to correspond directly to the physical-gauge path integral, we must modify the prescription for the path integral as follows: Choose an initial and a final configuration of the A_i. Now consider the families of configuration obtained from these by repeated application of a

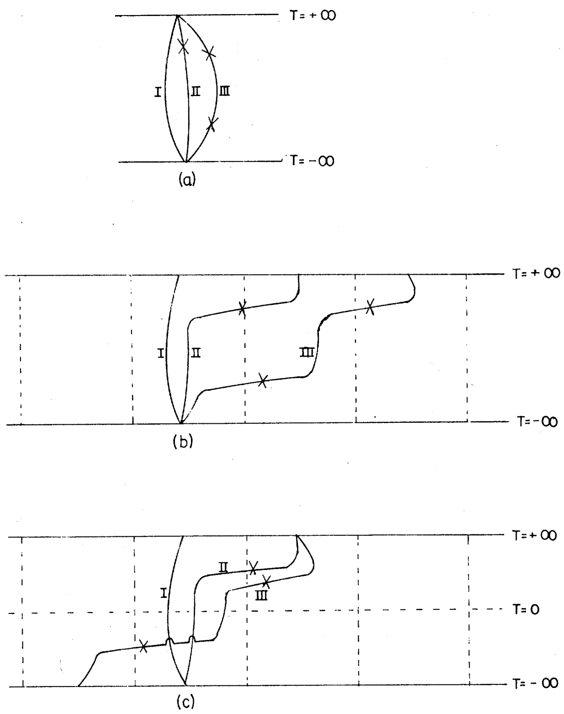

FIG. 1. (a) Schematic representation of three possible paths of the gauge field from $t=-\infty$ to $t=+\infty$ in a physical gauge. Crosses denote pseudoparticles. (b) The same three paths in the $A_0=0$ gauge. Vertical dashed lines separate sectors of different winding number. The freedom to make time-independent gauge transformations has been used to require that all paths start in the same sector at $t=-\infty$. (c) The paths in another $A_0=0$ gauge. Here, the available gauge freedom is used to specify the winding-number sector at $t=0$.

gauge transformation which changes the winding number by 1.[10] Integrate over all path running from any configuration in the initial family to any configuration in the final family, with the restriction that at some time t_0 (most conveniently taken to be the initial time) a further gauge condition hold; in particular, require that the path go through the sector with winding number n_0. In doing the integration, all paths should be weighted only by the action and not by any factor which depends on the initial or final winding number, since this is just the weighting used in physical gauges. In the language of JR and CDG, this prescription will pick out only the $\theta=0$ sector [or the $\theta=\beta$ sector if the term given in Eq. (6) is added], so there will be a unique vacuum.

These two methods of parametrizing the $A_0=0$ gauge function space are analogous to the two interpretations of the Lagrangian of Eq. (7). The former method, that of CDG and JR; is analogous to the periodic-potential interpretation; the states fall into different vacuum sectors. Using the latter method, analogous to the pendulum interpretation, there is a unique sector; by changing the Lagrangian it is possible to make the sector identical to any one sector of the former interpretation.

The possibility of two different parametrizations occurs because in the $A_0=0$ gauge the $F_{\mu\nu}$ do not uniquely determine the canonical variables; this phenomenon can also occur in other gauges, the most obvious being any gauge whose definition involves A_0. It will also occur if the gauge theory is treated using the method of Gervais, Sakita, and Wadia,[11] which introduces additional variables at the surface at infinity. Wadia and Yoneya[12] have shown how a many-vacuum description can arise in this context; by applying a method analogous to the second treatment of the $A_0=0$ gauge, it is also possible to obtain a single-vacuum description.

If one works in a physical gauge, where the canonical variables are uniquely determined, there is only one parametrization of function space possible; as we have seen, it corresponds to the pendulum interpretation of Eq. (7). The spectrum corresponds to that of only one of the sectors of the many-vacuum description. There is no observable distinction between the two descriptions since the physical gauge Lagrangian can be adjusted to correspond to any desired sector, in particular to the sector corresponding to the observed universe. Thus, the many vacuums are not essential to the understanding of pseudoparticles; rather, they are artifacts of a particular way of parametrizing the function space and thus of defining the path integral. It is quite possible, and in some gauges necessary, to do without them.

We are grateful to Norman Christ for many helpful conversations. Both of us would like to thank the Aspen Center for Physics and one of us (E.J.W.) would like to thank the Theory Group at the Stanford Linear Accelerator Center for their hospitality.

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*This research was supported in part by the U. S. Energy Research and Development Administration.

[1]G. 't Hooft, Phys. Rev. Lett. 37, 8 (1976).

[2]A. Belavin, A. Polyakov, A. Schwartz, and Yu. Tyupkin, Phys. Lett. 59B, 85 (1975).

[3]R. Jackiw and C. Rebbi, Phys. Rev. Lett. 37, 172 (1976).

[4]C. Callan, R. Dashen, and D. Gross, Phys. Lett. 63B, 334 (1976).

[5]More strictly, we define these to be gauges in which the number of pairs of canonical variables is equal to the number of degrees of freedom. The most obvious are

the Coulomb and axial gauges in electrodynamics and the axial gauge in Yang-Mills theory. Note that the specification of the gauge must completely determine the A_μ. Thus, in the axial gauge one must specify not only $A_3=0$, but also further conditions (e.g., $A_2=0$ if $z=z_0$, $A_1=0$ if $y=y_0$, $z=z_0$, and $A_0=0$ if $x=x_0$, $y=y_0$, $z=z_0$).

[6]Except for the discussion of the axial-vector current, our arguments hold whether or not the theory contains matter fields.

[7]J. Bell and R. Jackiw, Nuovo Cimento 60A, 47 (1969); S. Adler, Phys. Rev. 177, 2426 (1969).

[8]This was pointed out to us by Norman Christ.

[9]Strictly speaking, j_0 must contain no time derivatives, while j_i must be free of derivatives with respect to x_i and must approach zero at spatial infinity fast enough that the necessary integrations by parts are valid. All these conditions hold in the present case.

[10]In fact, these families should also include configurations obtained by localized gauge transformations (which do not change the winding number). This can be seen by comparison with the paths which enter the physical-gauge path integral. Alternatively, note that the imposition of the subsidiary condition requires that one work with states which are invariant under these gauge transformations, i.e., which are linear combinations (with no relative phase) of all such configurations.

[11]J. L. Gervais, B. Sakita, and S. Wadia, Phys. Lett. 63B, 55 (1976).

[12]S. Wadia and T. Yoneya, Phys. Lett. 66B, 341 (1977).